# Measuring Impoverishment: An Overlooked Dimension of Fiscal Incidence

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# Overview

- Standard measures of the effect of taxes and benefits on the poor
  - Poverty indicators (including squared poverty gap)
  - Inequality indicators
  - Stochastic dominance tests
  - Lorenz dominance tests
  - Measures of progressivity
  - Vertical and horizontal inequity
  - Anonymous social welfare comparisons

  ...leave out important information about how the poor are affected by fiscal policy.

# Overview

- For example, we can have:
  - First (and higher) order stochastic dominance
    - $\Rightarrow$ poverty (including the squared poverty gap) declining
    - $\Rightarrow$ generalized Lorenz dominance
  - Income distribution becoming less unequal
  - Progressive net taxes
  - Low or no horizontal inequity
  - But some of the poor become substantially poorer
- Extent to which a tax and transfer system impoverishes poor or makes non-poor people poor is valuable information for analyst and policymaker
- Formally define impoverishment and establish its relationship with traditional measures
- Propose using a Markovian transition matrix and an income loss matrix

# Definitions

- Well-being space $\Omega$
  - Could be income ($\Omega \subset \mathbb{R}_+$ and $\sup \Omega < \infty$) or multi-dimensional
- Income before taxes and transfers $y_i^0 \in \Omega$ and after taxes and transfers $y_i^1 \in \Omega$
- Cumulative distribution functions $F_0 : \Omega \to [0, 1]$ and $F_1 : \Omega \to [0, 1]$
- Poverty line $z \in \Omega$
- There is **impoverishment** if $y_i^1 < y_i^0$ and $y_i^1 < z$ for some $i$

# First Order Dominance

- Post-tax and transfer distribution does *not* weakly FOSD pre-tax and transfer distribution among the poor $\Rightarrow$ *impoverishment*

  - Proof: see paper.

- Post-tax distribution *does* weakly FOSD pre-tax distribution among the poor and tax-benefit system rank-preserving $\Rightarrow$ *no impoverishment*

  - Proof: see paper.

- Post-tax distribution *does* weakly FOSD pre-tax distribution among the poor and re-ranking $\Rightarrow$ dominance cannot determine impoverishment

  - Proof: $\boldsymbol{y^0} = (5, 8, 20), \boldsymbol{y^1} = (9, 6, 18), z = 10$. $F_1$ FOSD $F_0$ among the poor and there is impoverishment.

# Horizontal Equity and Progressivity

- Horizontal inequity is neither a necessary nor sufficient condition for impoverishment.
  - Not sufficient:
    $y^0 = (5, 5, 6, 20), y^1 = (5, 7, 6, 18), z = 10$. Horizontal inequity (classical and re-ranking) has occurred but impoverishment has not.
  - Not necessary:
    $y^0 = (5, 8, 20), y^1 = (6, 7, 20), z = 10$. Impoverishment has occurred but horizontal inequity (classical or re-ranking) has not.
- A progressive tax-benefit system is neither a necessary nor sufficient condition for no impoverishment.
  - Proof: see paper.

# Fiscal Mobility Matrix

- Directional mobility literature provides a useful framework
  - See, for example, Fields (2008)
- Compare the status of identified individuals in the before and after taxes and transfers situations
- One can see which individuals are adversely/favorably impacted by a particular policy
- We establish dominance criteria so that alternative policies can be compared in terms of the downward mobility they induce

# Definitions

- Fiscal Mobility
  - The directional movement between the before and after net taxes situations among $k$ pre-defined income categories
- Fiscal Mobility Matrix
  - $k \times k$ transition matrix $P$ where the $ij$-th element $p_{ij}$ is the probability of moving to income group $j$ after net taxes for an individual in group $i$ before net taxes
  - $\Rightarrow P$ is a stochastic matrix with $\sum_{j=1}^{k} p_{ij} = 1 \ \forall i \in \{1, \ldots, k\}$
- Poverty Lines
  - Let $\boldsymbol{z}$ be a vector of poverty lines between $\bar{z}$ and $\underline{z}$. These poverty lines determine a subset $r$ of the $k$ income categories ($r < k$) that are considered poor

# Downward Mobility Among the Poor

- If $\sum_{i=1}^{r} \sum_{j:j<i} p_{ij} > 0$, then there is downward mobility among the poor

- If $\sum_{i=r+1}^{k} \sum_{j:j<r} p_{ij} > 0$ then there is downward mobility of some non-poor into poverty

  - Example: $k = 6$ and $r = 3$



After Taxes and Transfers

# An Illustration: Brazil

- Income distribution after taxes and transfers
  <u>Lorenz dominates</u> distribution before taxes and transfers

  - $\Rightarrow$ Inequality unambiguously falls
  - To illustrate: Gini falls from 0.57 to 0.54
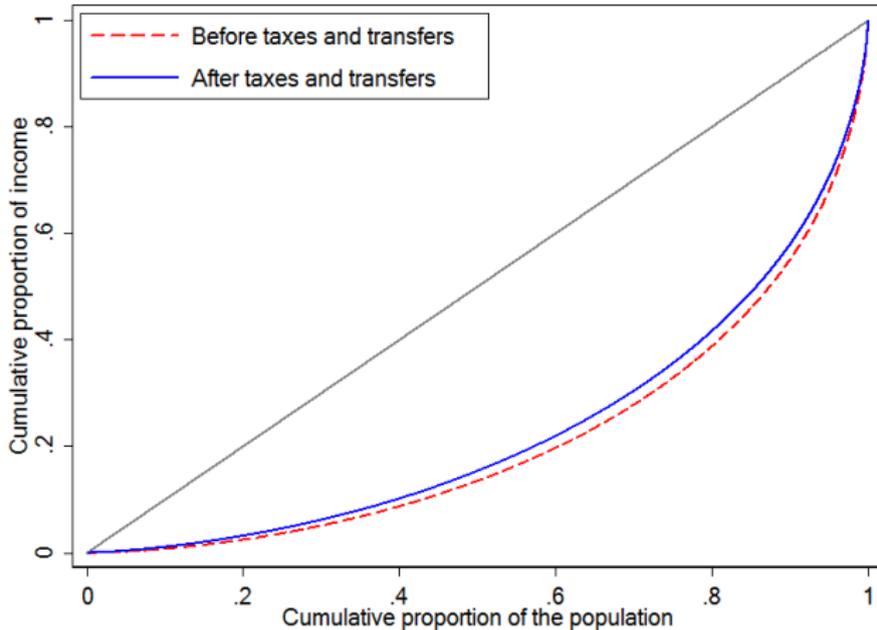
# An Illustration: Brazil

- First order stochastic dominance over domain of extreme poverty lines ($\leq$ $2.50 PPP per day)
- Second order stochastic dominance over domain of poverty lines ($\leq$ $4.00 PPP per day)
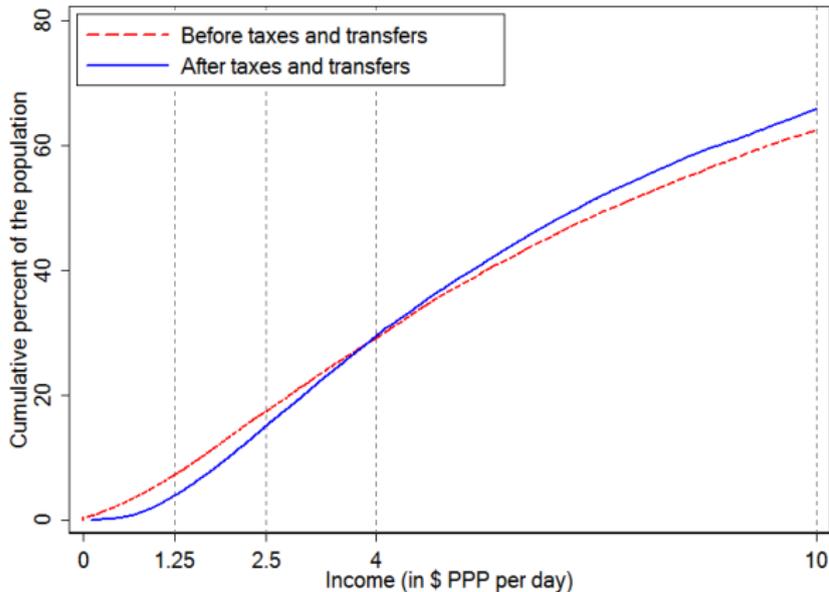
# An Illustration: Brazil

- Progressive overall tax system:
  - Kakwani index of direct and indirect taxes is 0.04
  - Kakwani index of direct transfers is 0.54
  - Reynolds-Smolensky index of after taxes and transfers income with respect to before taxes and transfers income is 0.05
- <u>However</u>:
  - 11.4% of the moderate poor become extreme poor
  - 10.5% of the vulnerable become poor

# Fiscal Mobility Matrix: Brazil

| | | Post-tax and transfer income groups | | | % of Pop. | Mean Income |
|---|---|---|---|---|---|---|
| | | < 2.50 | 2.50– 4.00 | 4.00– 10.00 | > 10.00 | |
| Pre-tax and transfer income groups | < 2.50 | 85% | 10% | 4% | 1% | 15.4% | $1.45 |
| | 2.50– 4.00 | 11% | 78% | 11% | 1% | 11.3% | $3.24 |
| | 4.00– 10.00 | 0% | 11% | 86% | 3% | 33.5% | $6.67 |
| | > 10.00 | 0% | 0% | 13% | 87% | 39.8% | $28.41 |
| | % of Pop. | 14.3% | 13.9% | 36.0% | 35.8% | 100% | $14.14 |

# How Much do the Losing Poor Lose?

- Matrix of average proportional losses
  - $k \times k$ matrix *L* with *ij*-th element $\ell_{ij}$ equal to the average percent decrease in income of those who began in group *i* and lost income due to taxes and transfers, ending in group $j \leq i$
  - Negative semi-definite and weakly lower-triangular by construction
  - There is impoverishment among the poor if and only if $\ell_{ij} < 0$ for some $j \leq r$

# Average Proportional Losses: Brazil

| | < 2.50 | 2.50– 4.00 | 4.00– 10.00 | > 10.00 | % of Pop. | Group Avg. |
|---|---|---|---|---|---|---|
| < 2.50 | −10% $1.93 | | | | 15.4% | −10% $1.93 |
| 2.50– 4.00 | −17% $2.72 | −11% $3.38 | | | 11.3% | −12% $3.28 |
| 4.00– 10.00 | | −18% $4.37 | −16% $7.03 | | 33.5% | −16% $6.70 |
| > 10.00 | | | −21% $11.02 | −21% $31.80 | 39.8% | −21% $28.85 |
| % of Pop. | 14.3% | 13.9% | 36.0% | 35.8% | 100% | |

Pre-tax and transfer income groups

# Fiscal Mobility Dominance

- In terms of fiscal mobility, is an alternative scenario more desirable *for the poor* than the actual scenario?

- Compare two fiscal mobility matrices $P$ and $P'$ and denote strong downward mobility dominance by the binary relation $\mathcal{M}$

- $P \mathcal{M} P'$ if $P$ exhibits less downward mobility among the poor (and into poverty) than $P'$

- Formally, $P \mathcal{M} P'$ if $\sum_{m=1}^{j} p_{im} \leq \sum_{m=1}^{j} p'_{im}$ for $i \in \{2, \ldots, k\}$ and $j \leq r < i$, with strict inequality for some $i$
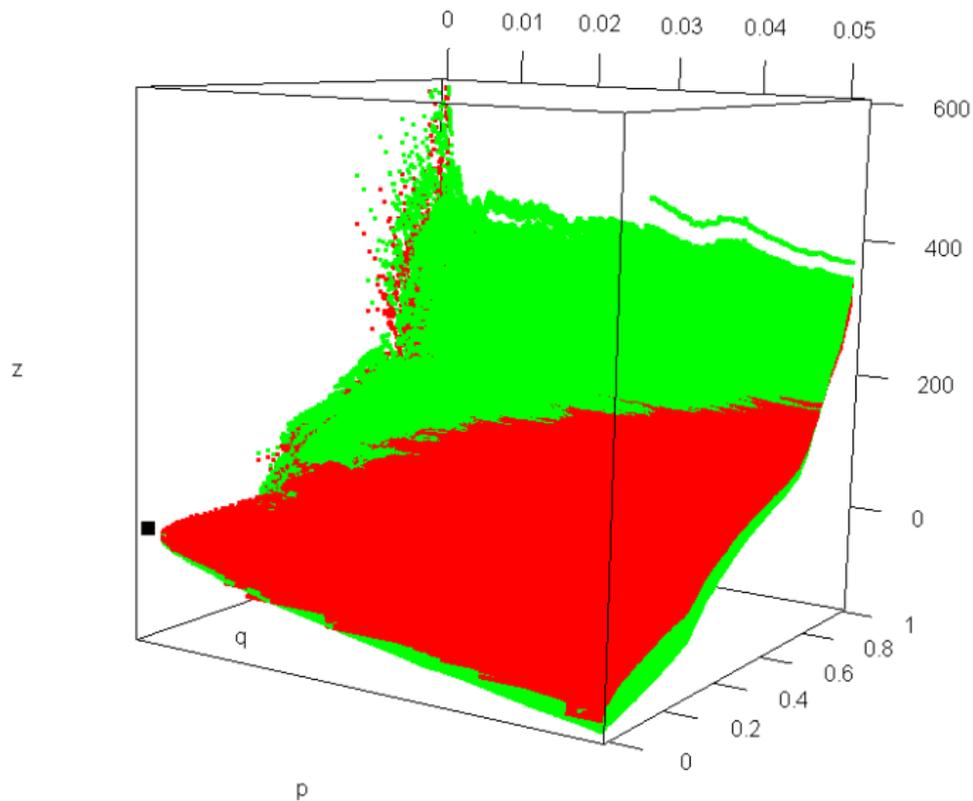
# Alternative Scenario: Neutral Tax

- Compare actual scenario in Brazil to an alternative
- Neutral (horizontally equitable) tax
  - Individuals are taxed proportional to their incomes such that total tax revenue remains fixed
- Transfers received are still as observed
- *Ex ante*, difficult to determine whether neutral tax system will entail more or less impoverishment than actual tax system
- 16% of moderate poor become extreme poor
- 15% of vulnerable become moderately poor

# Fiscal Mobility Matrix: Neutral Tax

|  | Post-tax and transfer income groups | | | | % of Pop. | Mean Income |
|---|---|---|---|---|---|---|
|  | < 2.50 | 2.50–4.00 | 4.00–10.00 | > 10.00 | | |
| < 2.50 | 85% | 10% | 4% | 1% | 15.4% | $1.45 |
| 2.50–4.00 | 16% | 73% | 10% | 1% | 11.3% | $3.24 |
| 4.00–10.00 | 0% | 15% | 82% | 3% | 33.5% | $6.67 |
| > 10.00 | 0% | 0% | 17% | 84% | 39.8% | $28.41 |
| % of Pop. | 14.8% | 14.6% | 35.9% | 34.7% | 100% | $14.14 |

Pre-tax and transfer income groups

# Bourguignon's Welfare Dominance

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