MEASURING IMPOVERISHMENT: AN OVERLOOKED DIMENSION OF FISCAL INCIDENCE

Sean Higgins and Nora Lustig

COMMITMENT TO EQUITY

ABSTRACT

The effect of taxes and benefits on the poor is usually measured using standard poverty and inequality indicators, stochastic dominance tests, and measures of progressivity and horizontal inequity. However, these measures can fail to capture an important aspect: that some of the poor are made poorer (or some of the non-poor made poor) by the tax-benefit system. We call this impoverishment and formally establish the relationships between impoverishment, stochastic dominance tests, horizontal inequity, and progressivity measures. The directional mobility literature provides a useful framework to measure impoverishment. We propose using a transition matrix and income loss matrix, and establish a mobility dominance criterion to compare alternate tax-benefit systems. We illustrate with data from Brazil.

Keywords: stochastic dominance, poverty, fiscal incidence, mobility

JEL: I32, H22

I.   INTRODUCTION

The effect of taxes and benefits on the poor is usually measured using standard poverty and inequality indicators, stochastic dominance tests, and measures of progressivity and vertical and horizontal inequity. Here we argue that the equity assessment of a tax and benefit system needs to incorporate another dimension: the extent of impoverishment induced by it. Stochastic dominance tests (Atkinson, 1987; Foster and Shorrocks, 1988) do not take into account individuals' initial position, so it is possible for poverty to unambiguously fall, while at the same time some pre-tax poor are further impoverished by the fiscal system. Thus, standard measures can fail to capture impoverishment caused by a tax and benefit system because decreases in income of some poor may be (more than) compensated by income increases of other poor.

We posit that the extent to which a tax system impoverishes the poor (or makes non-poor people poor) is valuable information for the analyst and the policymaker. Policymakers can use this information to modify government interventions or introduce new mechanisms that reduce impoverishment, if not completely eliminate it. In the next section, we formally define impoverishment and establish its relationship with traditional measures of inequality and poverty, horizontal inequity, stochastic dominance tests, and progressivity. In Section 3, we propose using a Markovian transition matrix in conjunction with an income loss matrix to assess the degree of impoverishment of a tax-benefit system. We also propose a partial ordering of downward mobility dominance based on the transition matrix. Although there are many measures of directional income mobility (see Fields (2008) for a survey) and mobility dominance (e.g., Fields et al., 2002), the results from the transition matrix (first used to measure income mobility in Champernowne (1953) and mobility induced by taxes in Atkinson (1980)) are straight-forward and easy to convey to policymakers. In Section 4, we use household survey data from Brazil to illustrate the failures of standard measures to capture impoverishment and to apply the transition and income loss matrices. Section 5 presents concluding remarks.

II.   IMPOVERISHMENT

The degree of impoverishment is completely overlooked by standard measures of inequality, poverty, and horizontal equity. Standard poverty and inequality measures are anonymous with respect to initial income, so impoverishment is not captured by these measures if the losses of some poor are compensated by gains of other even poorer individuals.

Furthermore, the degree of impoverishment is a different concept from the notion of horizontal inequity—a concept that by definition takes into account individuals' pre-tax position.[2] Even if some pre-tax poor are impoverished by the tax system, the ranking among the poor may not have changed (so there is no horizontal inequity due to re-ranking) and pre-tax equals may be impoverished to the same degree (so there

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[2] The view that the unequal treatment of equals—classical horizontal inequity—or re-ranking are unfair is accepted by a wide range of economists and philosophers, egalitarian or not. Horizontal inequity brings in the *status quo ante*; its measurement relies on comparisons that are "non-anonymous" in the language of Bourguignon (2011). Duclos (2008) reviews horizontal and vertical equity.

is no classical horizontal inequity—i.e., violation of the principle that pre-tax equals should be treated equally). Neither does the presence of horizontal inequity necessarily imply impoverishment, because there could be re-ranking among the poor or unequal treatment among pre-tax equals when the tax-benefit system lifts incomes of some of the poor without decreasing incomes of any poor (i.e., no impoverishment). Horizontal inequity is neither a necessary nor a sufficient condition for the presence of impoverishment of pre-tax poor (proposition 1). Thus, measures that account for horizontal inequity among the poor (e.g., Bibi and Duclos, 2007) will not necessarily capture this form of inequity of a tax system.

Dominance tests can also fail to capture impoverishment. Specifically, we have the following relationships between first order stochastic dominance and impoverishment. If the post-tax distribution does not weakly first order stochastic dominate the pre-tax distribution among the poor, impoverishment has occurred, regardless of whether re-ranking has occured (proposition 2). Is this a necessary condition as well? In other words, if the post-tax distribution weakly first order stochastic dominates the pre-tax distribution, does this imply the absence of impoverishment? If the post-tax distribution is rank preserving, yes (proposition 3). However, if there is re-ranking among the poor, first order stochastic dominance among the poor is not a sufficient condition for no impoverishment among the poor (proposition 4).

Formally, denote the well-being space Ω. For ease of exposition, we will take income as our measure of well-being, where income takes non-negative values and is bounded: $\Omega \subset \mathbb{R}_+$ and $\sup\{\Omega\} < \infty$. However, additional measures of well-being could easily be incorporated into the analysis to accommodate multi-dimensional poverty measures. Denote individual income before taxes and transfers by $y_i^0 \in \Omega$ and individual income after taxes and transfers by $y_i^1 \in \Omega$ for each $i \in S$ where S is the set of individuals in society. The set Ω can represent household per capita incomes or can account for differences in need among individuals (adjusting, for example, for different caloric needs based on age and economies of scale within households), in which case we assume there exists a function $\varphi: \mathbb{R}_+ \to \Omega$ which maps household per capita income into equivalized income. The cumulative distributions of the before and after taxes and transfers income concepts are non-decreasing functions $F_0: \Omega \to [0,1]$ and $F_1: \Omega \to [0,1]$.

We create the vectors $\boldsymbol{y^0}$ and $\boldsymbol{y^1}$ which contain as elements each individual's income before and after taxes and transfers, respectively.[3] In both vectors, individuals are ranked in ascending order of pre-tax income (in other words, if individual i occupies position k of $\boldsymbol{y^0}$, that same individual will also occupy position k of $\boldsymbol{y^1}$; if re-ranking occurs, the order of the $\boldsymbol{y^1}$ vector does not reflect this re-ranking). To be clear, the cumulative distribution function F_1 *does* re-rank individuals (unlike the vector $\boldsymbol{y^1}$): denoting $F(y) \equiv p$, if there is re-ranking caused by taxes and transfers, a certain value of p will not necessarily correspond to the

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[3] Note that the sum of the elements of $\boldsymbol{y^0}$ need not equal the sum of the elements of $\boldsymbol{y^1}$ for various reasons. Taxes could exceed transfers if tax revenues are spent on other things as well (e.g., defense). Transfers could exceed taxes if they are financed by other sources (e.g., oil revenues, debt). Finally, taxes could equal transfers in household per capita terms, but not in equivalized terms on Ω.

same individual in each distribution. The poverty line, which lies in the well-being space, will be denoted $z \in \Omega$.[4]

Definition 1. There is *impoverishment* if $y_i^1 < y_i^0$ and $y_i^1 < z$ for some individual i. In other words, the individual could have been poor before taxes and transfers and been made poorer by the fiscal system, or non-poor before taxes and transfers but poor after.

Definition 2. *Horizontal inequity* can be defined in two ways: the classical definition or the re-ranking definition. There is *classical horizontal inequity* if equals are treated unequally by the tax and transfer system. Classical horizontal inequity occurs if $y_i^0 = y_j^0$ and $y_i^1 \neq y_j^1$ for some pair of individuals (i, j) who, from an ethical viewpoint, should be treated equally by the fiscal system based on their characteristics. There is *re-ranking* if $y_i^0 \geq y_j^0$ and $y_i^1 < y_j^1$ for some such (i, j) pair. There is horizontal inequity *among the poor* if the above conditions hold and $y_k^n < z$ for all $k \in \{i, j\}$ and for some $n \in \{0,1\}$.

Definition 3. The post-tax and transfer income distribution F_1 weakly *first order stochastic dominates* the pre-tax and transfer income distribution F_0 if $F_1(y) \leq F_0(y)$ for all $y \in \Omega$. A less restrictive condition is that F_1 first order stochastic dominates F_0 *among the poor*, or $F_1(y) \leq F_0(y)$ for all $y \in [0, z]$. Note that, by the definition of cumulative distribution functions, first order stochastic dominance is an anonymous concept.

Proposition 1. Impoverishment does not imply horizontal inequity, and horizontal inequity among the poor does not imply impoverishment.

Proof. (a) Consider an example with three individuals where $\boldsymbol{y^0} = (5, 8, 20), \boldsymbol{y^1} = (6, 7, 20), z = 10$. Impoverishment has occurred, but neither classical horizontal inequity nor re-ranking has occurred. (b) Consider the example $\boldsymbol{y^0} = (5, 5, 6, 20), \boldsymbol{y^1} = (5, 7, 6, 18), z = 10$. Horizontal inequity among the poor has occurred (by both the classical and re-ranking definitions), but impoverishment has not.

Proposition 2. If F_1 does not weakly first order stochastic dominate F_0 among the poor, then impoverishment has occurred.

Proof. F_1 does not weakly first order stochastic dominate F_0 among the poor implies that there exists a $\hat{y} \in [0, z]$ such that $F_1(\hat{y}) > F_0(\hat{y})$. By the definition of cumulative distribution functions, this implies that the proportion of individuals with $y_i^1 < \hat{y}$ is higher than the proportion of individuals with $y_i^0 < \hat{y}$. Since the total number of individuals is identical in the pre-tax and post-tax distributions, this implies that there exists some individual j such that $y_j^0 > \hat{y}$ and $y_j^1 < \hat{y}$. Since $\hat{y} \leq z$, this implies $y_j^1 < y_j^0$ and $y_j^1 < z$, implying impoverishment has occurred.

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[4] Note that a single poverty line in Ω can still account for differences in needs across individuals, since household per capita incomes are mapped into equivalized incomes in Ω by the function φ.

Proposition 3. If there is no re-ranking among the poor and F_1 first order stochastic dominates F_0 among the poor, then impoverishment has not occurred.

Proof. By contrapositive: impoverishment among the poor implies that $y_i^1 < y_i^0$ and $y_i^1 < z$ for some individual i. If there does not exist an individual j with $y_j^0 < y_i^0 \leq y_j^1$ then $F_1(y_i^1) > F_0(y_i^1)$ which implies F_1 does not first order stochastic dominate F_0 on the interval $[0, z]$. If there does exist such an individual j, then re-ranking has occurred.

In practice, tax and benefit systems entail some degree of re-ranking among the poor. In this case, proposition 4 tells us that we cannot rely on first order stochastic dominance to indicate that there has been no impoverishment.

Proposition 4. If there is re-ranking among the poor, first order stochastic dominance of F_1 over F_0 among the poor is not a sufficient condition for the absence of impoverishment.

Proof. Consider the example where $y^0 = (5, 8, 20), y^1 = (9, 6, 18), z = 10$. F_1 first order stochastic dominates F_0 among the poor and there is impoverishment.

Since first order stochastic dominance implies higher order dominance, it also tells us that we cannot use second order dominance to conclude that there has been no impoverishment. Furthermore, since first order stochastic dominance among the poor implies an unambiguous reduction in poverty according to any poverty measure in a broad class of additively separable measures[5] (Atkinson, 1987), this also tells us that poverty measures will not necessarily capture impoverishment. (This fact can also be seen with simple examples; consider the example in the proof of proposition 4.)

Inequality measures are similarly anonymous with respect to initial income and are therefore also likely to overlook impoverishment. First order stochastic dominance implies second order stochastic dominance, which is equivalent to generalized Lorenz dominance (Foster and Shorrocks, 1988). When $\int_\Omega y^0 dF_0(y) = \int_\Omega y^1 dF_1(y)$ (i.e., the distributions have equal means), generalized Lorenz dominance implies Lorenz dominance which implies a lower Gini coefficient. Since first order stochastic dominance is not a sufficient condition for the absence of impoverishment (provided that re-ranking has occurred), it follows that the Gini coefficient and Lorenz dominance tests can show unambiguously lower inequality in spite of impoverishment.

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[5] Specifically, let the poverty function $p(y, z)$ be defined on all of $\Omega \times \Omega$ with $p(y, z) = 0$ whenever $y \geq z$. First order stochastic dominance implies a reduction in poverty using any poverty measure from the class of additively separable measures P such that there exists a monotonic transformation $G(P) = \int_\Omega p(y, z) dF(y)$ satisfying $G'(P) < 0$ (Atkinson 1987). This class of poverty measures includes the headcount, poverty gap, and squared poverty gap indices (indeed, it includes any member of the class of poverty measures proposed by Foster et al. (1984)), the Watts (1968) measure, and the second measure proposed by Clark et al. (1981).

It is worth noting that the traditional results with respect to generalized Lorenz dominance implying higher social welfare[6] (Shorrocks, 1983) use anonymous social welfare functions: individual utilities and the social welfare function for the post-tax and transfer distribution are independent of what the pre-tax and transfer income distribution would have looked like, how much different individuals gained or lost from the tax-benefit system, and whether some of the individuals who lost were already unable to afford basic necessities. If individual utility functions or social welfare are dependent on the amount individuals gain or lose (as in Bourguignon, 2011), the traditional results no longer hold. Furthermore, if individuals are loss averse (Kahneman and Tversky, 1979), it is easy to show that a transfer satisfying the Pigou-Dalton transfer principle from a poor individual to a poorer one can lead to not only impoverishment but also lower social welfare.

Standard measures of progressivity and redistributive effect, despite being non-anonymous with respect to income before taxes and transfers, can indicate that a tax-benefit system is progressive even when it impoverishes a substantial proportion of the poor. We illustrate this with a common progressivity indicator from Kakwani (1977) and a common measure of redistributive effect from Reynolds and Smolensky (1977).

Definition 4. Suppose that the population is large enough that F_0 and F_1 can be approximated by continuous and differentiable functions. Suppose they are also invertible so that $F_0(y) = p$ implies $y = F_0^{-1}(p)$. The *Lorenz curve of pre-tax and transfer income* is $L_0(p) \equiv \int_0^{F_0^{-1}(p)} y^0 dF_0(y) / \mu^0$ where $\mu^0 \equiv \int_\Omega y^0 dF_0(y)$ is mean income before taxes and transfers. The *Gini coefficient of pre-tax and transfer income* is defined as $G_0 \equiv 1 - 2 \int_0^1 L_0(p) dp$

Definition 5. The *concentration curve of post-tax and transfer income with respect to pre-tax and transfer income* is $c_1(p) \equiv \int_0^{F_0^{-1}(p)} y^1 dF_0(y) / \mu^1$ where $\mu^1 \equiv \int_\Omega y^1 dF_1(y)$ is mean income after taxes and transfers. The *concentration index of post-tax and transfer income with respect to pre-tax and transfer income* is $C_1 \equiv 1 - 2 \int_0^1 c_1(p) dp$.

Definition 6. Denote the taxes paid (benefits received) by individual i as t_i (b_i). By definition, $y_i^1 = y_i^0 - t_i + b_i$. Denote total taxes paid (benefits received) as a fraction of total pre-tax and transfer income in society as $\bar{t}$ $(\bar{b})$. The *concentration curve of taxes* is $c_t(p) \equiv \int_0^{F_0^{-1}(p)} t dF_0(y) / \bar{t}\mu^0$ and the *concentration index of taxes* is $C_t \equiv 1 - 2 \int_0^1 c_t(p) dp$. The concentration curve and index of benefits are defined analogously.

Definition 7. The *Reynolds-Smolensky index of post-tax and transfer income with respect to pre-tax and transfer income* is $R \equiv 2 \int_0^1 c_1(p) - L_0(p) dp = G_0 - C_1$. The fiscal system is progressive if $R \in (0, G_0]$.

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[6] Provided that the social welfare function is non-decreasing and S-concave, and utility functions are concave, increasing, and identical for all individuals.

Definition 8. The *Kakwani index of taxes* is $K_t \equiv 2 \int_0^1 L_0(p) - c_t(p)dp = C_t - G_0$ and the *Kakwani index of transfers* is $K_b \equiv 2 \int_0^1 c_b(p) - L_0(p)dp = G_0 - C_b$. Taxes are progressive if $K_t \in (0, 1 - G_0]$ and benefits are progressive if $K_b \in (0, 1 + G_0]$.[7]

Proposition 5. A progressive tax-benefit system is neither a necessary nor sufficient condition for no impoverishment.

Proof. Not sufficient: consider the example where $\boldsymbol{y^0} = (5, 8, 20), \boldsymbol{y^1} = (9, 6, 14), z = 10$, taxes are $\boldsymbol{t} = (1, 4, 7)$ and benefits $\boldsymbol{b} = (5, 2, 1)$. We have $R = 0.28, K_t = 0.05, K_b = 0.95$, which indicates a progressive net fiscal system, but impoverishment has occurred. Not necessary: consider an example with no impoverishment where $\boldsymbol{y^0} = (8, 13, 20), \boldsymbol{y^1} = (8, 11, 22), z = 10, \boldsymbol{t} = (1, 3, 1), \boldsymbol{b} = (1, 1, 3)$. We have $R = -0.05, K_t = -0.29, K_b = -0.11$, indicating a regressive net fiscal system.

III.   MEASURING IMPOVERISHMENT: THE FISCAL MOBILITY MATRIX

The directional mobility literature provides a useful framework to measure impoverishment and convey this information to policymakers.[8] We define two matrices which, despite their shortcomings, provide a useful first assessment of impoverishment and present information in a way that is easy to convey to policymakers.[9]

Fiscal mobility is the directional movement between the before and after net taxes situations among k pre-defined income categories. It can be represented by the $k \times k$ transition matrix P, where the ijth element of P, denoted p_{ij}, can be interpreted as the probability of moving to income group j after taxes and transfers for individuals who were in income group i before taxes and transfers. Hence, P is a row stochastic matrix with $\sum_{j=1}^{k} p_{ij} = 1$ for all $i \in \{1, \dots, k\}$. Transition matrices were first used to compare pre- and post-tax income distributions in Atkinson (1980).

Define $\boldsymbol{z}$ as a vector of poverty lines between $\underline{z}$ (the lowest reasonable poverty line) and $\bar{z}$ (the highest reasonable poverty line). In other words, $\boldsymbol{z}$ is an ordered vector whose component values define tranches of income ranges which demarcate varying degrees of poverty severity.[10] These poverty lines will determine a subset r of the k income categories ($r < k$) for which p_{ij} denotes the probability of moving into more severe poverty (poverty) after net taxes, for individuals who were less poor (not poor) before net taxes. For

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[7] Kakwani (1977) proposed an index of progressivity for taxes. Lambert (1985) proposed a Kakwani index for transfers analogous to the negative of the Kakwani index for taxes, so that a positive Kakwani index implies progressivity in both cases. Attempting to define a Kakwani index for the net fiscal system is problematic because net taxes (i.e., taxes minus benefits) are positive for some individuals and negative for others, which creates a concentration curve of net taxes that is not well-behaved (Lambert, 2001).

[8] Directional mobility is a subcategory of the "mobility as movement" definition (as opposed to the time independence definition). See Fields (2008) for a survey of the income mobility literature.

[9] For a discussion of the drawbacks of using transition matrices to measure mobility, see Foster and Rothbaum (2012), who also list a number of axioms for mobility measures and propose a more sophisticated measure.

[10] We are grateful to Peter Lambert for suggesting this interpretation of the vector of poverty lines.

example, we could let $k = 4, r = 2$, where the income groups are extreme poor, moderate poor, near poor, and non-poor. If $\sum_{i=1}^{r}\sum_{j:j<i}p_{ij} > 0$, then there is downward mobility among the poor. If $\sum_{i=r+1}^{k}\sum_{j:j<r}p_{ij} > 0$ then there is downward mobility of some non-poor into poverty.

The fiscal mobility matrix can provide us with a useful framework for answering the following question: in terms of fiscal mobility, is an alternative tax-benefit system more desirable for the poor than the actual scenario? We define downward mobility dominance, denoted by the relation $\mathcal{M}$, as follows. Situation P $\mathcal{M}$ P' if P exhibits less downward mobility among the poor (and from the non-poor into poverty) relative to P'. Formally, downward mobility dominance means that $\sum_{j=1}^{l}p_{ij} \leq \sum_{j=1}^{l}p'_{ij}$ for all i and l such that $l \leq r < i$, with strict inequality for some i.

Note that downward mobility does not capture the impoverishment of poor individuals who remain in their income group. Moreover, we are interested in knowing not only what percentage of the poor (non-poor) becomes poorer (poor) but also how much they lose on average. Let L be the $k \times k$ matrix of proportional income losses, with element l_{ij} equal to the average percent decrease in income of those who began in group i and lost income due to taxes and transfers, ending in group $j \leq i$. By construction, L is negative semidefinite and weakly lower-triangular. There is impoverishment if and only if $l_{ij} < 0$ for some $j \leq r$.

IV. AN ILLUSTRATION WITH BRAZILIAN DATA

In this section we use Brazilian data to show how standard poverty and inequality indicators, stochastic dominance tests, and measures of progressivity would lead us to conclude that Brazil's tax and transfer system is overall favorable to the poor. However, our conclusions may be less favorable when impoverishment and fiscal mobility are taken into account. The analysis uses the *Pesquisa de Orçamentos Familiares* (Family Expenditure Survey; POF) 2008-2009. We compare market income (before taxes and transfers) to post-fiscal income (after direct and indirect taxes and direct cash transfers). [11]

The Brazilian data illustrates many of the points made in section 2. The after taxes and transfers income distribution Lorenz dominates the before taxes and transfers distribution, so inequality unambiguously falls. (To illustrate, the Gini falls from 0.57 before taxes and transfers to 0.54 after.) The post-tax distribution first order stochastic dominates the pre-tax distribution on the interval between zero and slightly above $3 PPP per day, so poverty unambiguously falls below this income level (Figure 1). (To illustrate, the headcount index at $2.50 PPP per day falls from 15.4% to 14.3%, and the squared poverty gap from 3.8% to 2.3%.)

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[11] We follow the methodology and use the income concepts proposed by Lustig and Higgins (2013). For more details about the specific methodology used for the Brazilian data, see Higgins and Pereira (2013). Note that the framework presented here can be applied to two types of data: data in which actual taxes and benefits are enumerated, or data in which they are computed from a tax-benefit microsimulation model. In the data set we use for illustration, direct taxes and transfers are directly reported in the survey. Indirect (consumption) taxes are imputed using consumption data.

Furthermore, the post-tax distribution second order stochastic dominates the pre-tax distribution on the entire interval of reasonable poverty lines from zero to $4 PPP per day, which implies (Atkinson, 1987) an unambiguous reduction in poverty using any measure of the form given in footnote 3 where $p(y,z)$ is continuous, non-decreasing, and weakly concave in y.[12] Poverty measures of this class will all show a reduction in poverty for any reasonable poverty line, and will thus mask significant impoverishment among those who live on less than $4 PPP per day, which will be observed in the fiscal mobility and income loss matrices.

FIGURE 1. CUMULATIVE DISTRIBUTION FUNCTIONS BEFORE AND AFTER TAXES AND TRANSFERS IN BRAZIL
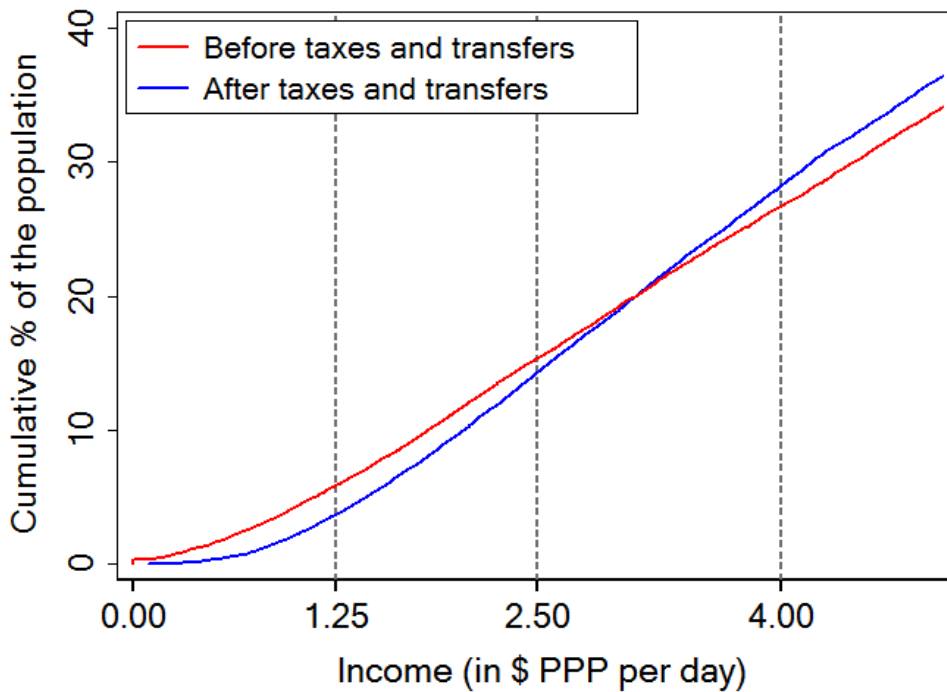
Source: Authors' calculations based on POF (2008-2009).

Common progressivity indicators also indicate that the tax-benefit system is progressive: we have $R = 0.05, K_t = 0.04, K_b = 0.54$. Note that these indices are calculated with respect to the before tax and transfer income distribution and are thus non-anonymous. However, they still mask impoverishment.

Table 4 shows the fiscal mobility matrix P for Brazil; the added row (column) labeled "percent of population" give population shares for the market income (post-fiscal income) groups, while the last column gives the mean market income (in purchasing-power parity adjusted US dollars per day) of members of that market income group. Our income groups in this example are four in total. The poor are divided into two

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[12] These additional restrictions on the poverty function preclude the headcount index, but do not preclude any of the other measures mentioned in footnote 3, such as the poverty gap index and squared poverty gap index.

groups: those with less than $2.50 PPP per day (the extreme poor) and between $2.50 and $4 PPP per day (the moderately poor). The two non-poor groups are: between $4 and $10 PPP per day (the vulnerable) and above $10 PPP per day.[13] As a result of (mainly indirect) taxes, 10.6% of those vulnerable to poverty become poor and 11.4% percent of the moderate poor become extremely poor. As noted above, this downward mobility is not captured by the standard measures of inequality, poverty, progressivity, and incidence.

TABLE 2. FISCAL MOBILITY MATRIX FOR BRAZIL

		Post-tax and transfer income groups					
		<2.5	2.5-4	4-10	>10	Percent of population	Mean income
Pre-tax and transfer income groups	<2.5	84.7%	10.3%	3.8%	1.3%	15.4%	$1.45
	2.5-4	11.4%	77.5%	10.5%	0.6%	11.3%	$3.24
	4-10	0.0%	10.5%	86.2%	3.2%	33.5%	$6.67
	>10	0.0%	0.0%	13.4%	86.6%	39.8%	$28.41
	Percent of population	14.3%	13.9%	36.0%	35.8%	100.0%	$14.14

Note: Mean incomes are measured in pre-tax income and are in US$ PPP per day. Rows may not sum to exactly 100% due to rounding. Differences in group shares between the before and after taxes and transfers distributions are all statistically significant from zero at the 0.1% significance level.
Source: Authors' calculations based on POF (2008-2009).

Now that we have established that taxes and transfers induce downward mobility among the poor, the next step is to ask *how much* the impoverished lose. For this, we use the income loss matrix *L*, shown in Table 5. The income loss matrix shows us the average loss of losers, by their pre- and post-taxes and transfers income groups, as a proportion of their before taxes and transfers incomes. In addition, we include the average before taxes and transfers incomes of each of these groups below the percent income loss. The last column shows the average income loss and market income of everyone who paid more taxes than they received benefits in the corresponding market income group. The extreme poor who are impoverished have before transfers income of $1.93 PPP per day on average and lose 9.6% of their income on average. The moderately poor who become extremely poor have before transfers income of $2.72 PPP per day and lose 16.8% of their income on average.

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[13] The $2.50 and $4 PPP per day poverty lines are commonly used as extreme and moderate poverty lines for Latin America, and roughly correspond to the median official extreme and moderate poverty lines in those countries (CEDLAS and World Bank, 2010). The $10 PPP per day line is the upper bound of those vulnerable to falling into poverty in three Latin American countries, calculated by Lopez-Calva and Ortiz-Juarez (2013) and the lower bound of the middle class used by Kharas (2010) and Ferreira et al. (2013).

TABLE 3. INCOME LOSS MATRIX FOR BRAZIL

		Post-tax and transfer income groups					
		<2.5	2.5-4	4-10	>10	Percent of population	Group average
Pre-tax and transfer income groups	<2.5	-9.6% $1.93				15.4%	-9.6% $1.93
	2.5-4	-16.8% $2.72	-10.7% $3.38			11.3%	-11.7% $3.28
	4-10		-18.1% $4.37	-15.8% $7.03		33.5%	-16.2% $6.70
	>10			-20.6% $11.02	-20.5% $31.80	39.8%	-20.5% $28.85
	Percent of population	14.3%	13.9%	36.0%	35.8%	100.0%	

Note: All monetary amounts are measured in pre-tax income and are in PPP-adjusted dollars per day. Zeroes are omitted from the matrix for enhanced readability. Differences in group shares between the before and after taxes and transfers distributions are all statistically significant from zero at the 0.1% significance level.
Source: Authors' calculations based on POF (2008-2009).

To illustrate the concept of downward mobility dominance, we compare the actual fiscal system to an alternative system in which transfers received are still as observed in our data, while the current (progressive) tax system is replaced by a neutral tax system that generates the same amount of tax revenue as the current system. As before, if we denote the total taxes collected divided by total pre-tax and transfer income by $\bar{t}$, everyone pays taxes proportional to their income at rate $\bar{t}$ in the neutral tax system. Hence, the neutral tax system is horizontally equitable and neither progressive nor regressive. *Ex ante*, it is difficult to determine whether the neutral tax system will entail more or less impoverishment than the actual tax system. Table 3 shows the fiscal mobility matrix where post-tax and transfer income is calculated assuming the neutral tax system instead of the actual tax system.

TABLE 4. FISCAL MOBILITY MATRIX FOR BRAZIL UNDER NEUTRAL TAX SYSTEM

		Post-tax and transfer income groups					
		<2.5	2.5-4	4-10	>10	Percent of population	Mean income
Pre-tax and transfer income groups	<2.5	84.6%	9.5%	4.4%	1.4%	15.4%	$1.45
	2.5-4	16.1%	72.9%	9.9%	1.1%	11.3%	$3.24
	4-10	0.0%	14.5%	82.1%	3.4%	33.5%	$6.67

	>10	0.0%	0.0%	16.5%	83.5%	39.8%	$28.41
	Percent of population	14.8%	14.6%	35.9%	34.7%	100.0%	$14.14

Note: Mean incomes are measured in pre-tax income and are in US$ PPP per day. Rows may not sum to exactly 100% due to rounding. Differences in group shares between the before and after taxes and transfers distributions are all statistically significant from zero at the 0.1% significance level.
Source: Authors' calculations based on POF (2008-2009).

Applying the downward mobility dominance relation $\mathcal{M}$ from section 2, it is easy to see that the fiscal system resulting from the actual tax structure downward mobility dominates (i.e., exhibits less downward mobility among the poor and from the non-poor into poverty) a neutral tax system. The two post-tax and transfer distributions could also be compared using Bourguignon's (2011) "general social welfare dominance criterion when the status quo enters individual utility functions," where pre-tax and transfer income is treated as the status quo.[14] This dominance relation (like the relation $\mathcal{M}$) is only a partial ordering, but there are many practical scenarios—including the comparison of the actual fiscal system to a neutral tax in Brazil—where neither distribution dominates the other (even if we restrict the analysis to the domain of the poor) according to Bourguignon's criterion.

V.   CONCLUDING REMARKS

We have shown that a country can perform well by standard indicators of inequality, poverty, Lorenz dominance, first order stochastic dominance, and progressivity despite having impoverishment and a non-trivial sub-section of the poor experience downward fiscal mobility into a lower income group (and having a non-trivial sub-section of the non-poor experience downward mobility into poverty). Standard indicators, such as the Gini, headcount, poverty gap, and squared poverty gap indices, as well as dominance criteria using Lorenz curves and cumulative distribution functions, overlook impoverishment because they do not concern themselves with who the before transfers poor are. Non-anonymous indicators of progressivity also overlook impoverishment.

The relationship between first order stochastic dominance, re-ranking, and impoverishment can be summarized as follows. If the post-tax and transfer distribution does not weakly first order stochastic dominate the pre-tax and transfer distribution, then impoverishment has occurred. If, on the other hand, it does dominate, one must check whether re-ranking occurred. If the tax-benefit system was rank-preserving and the post-tax distribution first order stochastic dominates the pre-tax distribution, no impoverishment has occurred. If, however, re-ranking took place, dominance tests cannot be used to determine whether there was impoverishment. In this case, the fiscal mobility matrix and income loss matrix can be used to determine if impoverishment occurred and measure it.

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[14] The status quo in Bourguignon's framework is usually post-tax and transfer income before some proposed reform to the fiscal system, against which post-tax and transfer income under two potential reforms are compared. However, he also mentions its applicability to the scenario to which we are applying it here, where a planner is comparing two distributions on their distance from the market income distribution.

Fiscal mobility matrices are a useful tool for identifying how much downward fiscal mobility occurs among the poor. In the case of Brazil we saw that 11% of the vulnerable become poor and 11% of the moderate poor become extremely poor despite any cash transfers they receive. Some of those who begin in the extremely poor group are also impoverished, losing 10% of their already low incomes on average. Meanwhile, we would not have been aware of this impoverishment and downward fiscal mobility if we relied on standard tools; extreme poverty and inequality decline, there is first order stochastic dominance to the left of $3 PPP per day, second order stochastic dominance over the domain of reasonable poverty lines, and taxes and transfers are progressive.

Although here we apply the notion of impoverishment to assess tax and benefit systems, its usage can be extended to any "before-after" situation. For example, it can be applied to analyze the changes in trade policy, rising food prices, a depreciation of the currency, or fiscal austerity measures.

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The CEQ logo is a stylized graphical representation of a Lorenz curve for a fairly unequal distribution of income (the bottom part of the C, below the diagonal) and a concentration curve for a very progressive transfer (the top part of the C).