The Analytics of Fiscal Redistribution

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Fiscal Policy and Inequality
Four Key Questions

- Does the net fiscal system decrease inequality?
- Is a particular tax or transfer equalizing or unequalizing?
- What is the contribution of a particular tax or transfer (or any combination of them) to the change in inequality?
- What is the inequality impact if one increases the size of a tax (transfer) or its progressivity?

Chapter Outline

- **Fiscal Redistribution: Single and Multiple Interventions**
- Allowing for Reranking
- Allowing for No Dominance
- Allowing for Different Original Distributions
- Different Inequality Measures
- Poverty

Assumptions for Now

- **No reranking:** the ordering of individuals in the post-fiscal state is the same as in the pre-fiscal state: i.e., no swapping of places
- **Dominance:** pre-fiscal and post-fiscal Lorenz curves do not cross (and the difference is statistically significant)
- **Same pre-fiscal (original) income distribution:** rules out comparisons of redistributive of fiscal systems across countries and over-time

Key questions addressed for the following cases

- Single intervention system:
  - Tax OR
  - Transfer
- Multiple interventions system
  - One tax and one transfer
  - *n* taxes and *m* transfers

➢ Lambert's conundrum and the startling consequences of path dependency

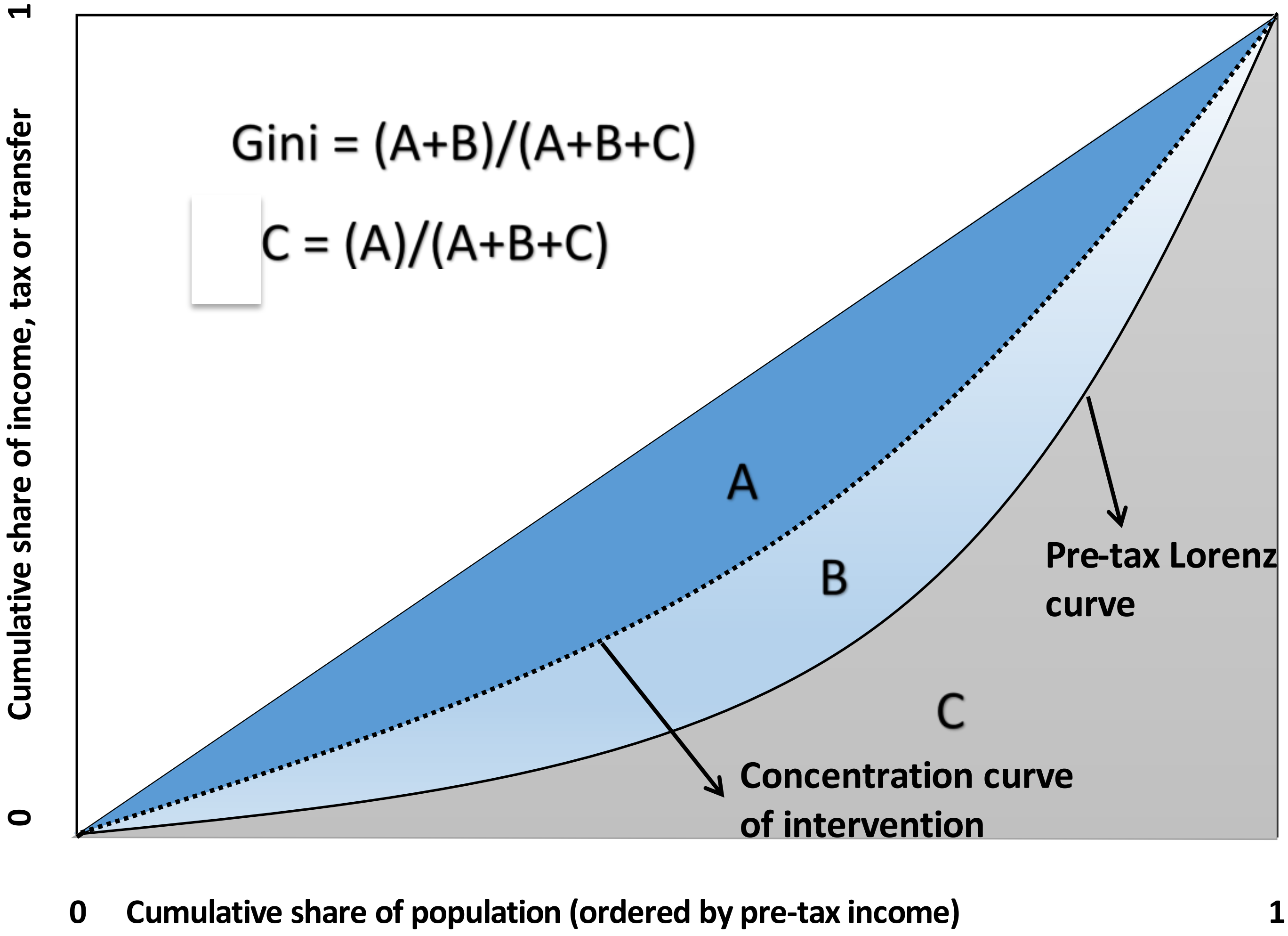
Fiscal System with a Single Intervention

Single Intervention

- *Single* can mean that all the taxes are added into a single category (same for transfers)

- Progressivity measures
  - Concentration curve
  - Concentration coefficient
  - Kakwani Index

Concentration Coefficient: C



Kakwani Index

- Progressive Tax: $\prod_T^K = C_t - G_x > 0$
- Proportional Tax: $\prod_T^K = C_t - G_x = 0$
- Regressive Tax: $\prod_T^K = C_t - G_x < 0$

Impact on Inequality Depends On…

- Progressivity of a tax (transfer)
- Size of the tax (transfer), where size equals the total tax (transfer) divided by total pre-tax (pre-transfer) income
  - ➢ A large regressive tax can be more equalizing than a small progressive one

Fiscal Policy and Inequality
Four Key Questions

- Does the net fiscal system decrease inequality?
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- What is the inequality impact if one increases the size of a tax (transfer) or its progressivity?

Progressivity vs. Size of Intervention: A System with Only One Tax

- In a system with only one tax:

$$RE_T = \frac{g}{1-g}\prod_T^K$$

- Getting the partial derivatives:

$$\frac{\partial RE_T}{\partial g} = \frac{1}{(1-g)^2}\prod_T^K$$

$$\frac{\partial RE_T}{\partial \prod_T^K} = \frac{g}{1-g}$$

Fiscal System with Multiple Interventions

Fiscal Policy and Inequality
Four Key Questions

- Does the net fiscal system decrease inequality?
- Is a particular tax or transfer equalizing or unequalizing?
- What is the contribution of a particular tax or transfer (or any combination of them) to the change in inequality?
- What is the inequality impact if one increases the size of a tax (transfer) or its progressivity?

Does the net fiscal system decrease inequality?

Let’s define the Redistributive Effect of the net fiscal system as

$$\boldsymbol{RE_N = G_x - G_N}$$

Where $\boldsymbol{G_x}$ $\boldsymbol{and}$ $\boldsymbol{G_N}$ are the pre-tax-pre-transfer Gini coefficient and post-tax-post-transfer Gini, respectively

Does the net fiscal system decrease inequality?

From Lambert (2001), we know that $\boldsymbol{RE_N}$ is equal to the weighted sum of the redistributive effect of taxes and transfers

$$\boldsymbol{RE_N = \frac{(1-g)RE_t + (1+b)RE_B}{1-g+b}}$$

Where

- $\boldsymbol{RE_t}$ and $\boldsymbol{RE_B}$ are the Redistributive Effect of the tax and the transfer, respectively
- **g** and **b**: size of tax and transfer, respectively.
  That is, total taxes and total transfers divided by total pre-tax and pre-transfer income, respectively

Does the net fiscal system decrease inequality?

For the net fiscal system to be equalizing:

$$\boldsymbol{RE_N = \frac{(1-g)RE_t+(1+b)RE_B}{1-g+b} > 0}$$

Condition 1:

$$\boldsymbol{\rightarrow RE_t > -\frac{(1+b)}{(1-g)}RE_B}$$

Does the net fiscal system decrease inequality?

		Transfer		
		Regressive $\rho_B^K < 0$	**Neutral** $\rho_B^K = \mathbf{0}$	**Progressive** $\rho_B^K > 0$
Tax	**Regressive** $\Pi_T^K < 0$	Always Unequalizing	Always Unequalizing	Equalizing if and only if Condition 1 holds
	Neutral $\Pi_T^K = \mathbf{0}$	Always Unequalizing	No Change in Equality	Always Equalizing
	Progressive $\Pi_T^K > 0$	Equalizing if and only if Condition 1 holds	Always Equalizing	Always Equalizing

Condition 1:

$$\rightarrow RE_t > -\frac{(1+b)}{(1-g)} RE_B$$

- The above result is well-known in the literature:
  - A fiscal system with a regressive tax can be equalizing as long as transfers are progressive and the condition above is fulfilled
  - A fiscal system with a regressive tax that collects more revenues than a less regressive one may be more equalizing
- However, Lambert's equation has more fundamental implications

Fiscal Policy and Inequality
Four Key Questions

- Does the net fiscal system decrease inequality?
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Is a particular tax or transfer equalizing?

- If there is a single intervention in the system, any of the progressivity measures discussed earlier will give an unambiguous answer

- If there is a tax **and** a transfer, then this is no longer the case
  - A regressive tax can be equalizing in the sense that the reduction in inequality can be larger with the tax than without it

Lambert's Conundrum

	1	2	3	4	Total
Original Income x	10	20	30	40	100
Tax *t*	6	9	12	15	42
Transfer *B*	21	14	7	0	42
Net Income *N*	25	25	25	25	100

Source: Lambert, 2001, Table 11.1, p. 278

Lambert's Conundrum

- The Redistributive Effect of the tax only in this example is equal to -0.05, highlighting its regressivity

- The Redistributive Effect of the transfer is equal to 0.19

➢ Yet, the Redistributive Effect of the net fiscal system is 0.25, higher than the effect without the taxes!

Lambert’s Conundrum

	1	2	3	4	Total
Original Income x	10	20	30	40	100
Transfer *B*	21	14	7	0	42
Post-Transfer Income	31	34	37	40	142
Tax *t*	6	9	12	15	42
Net Income *N*	25	25	25	25	100

Source: Lambert, 2001, Table 11.1, p. 278

Lambert's Conundrum
Path Dependency

- If a tax is regressive vis-à-vis the original income but progressive with respect to the less unequally distributed post-transfer income

- Regressive taxes *can* exert an equalizing effect over an above the effect of progressive transfers

- Note that institutional path dependency is not the same as mathematical path dependency

When could a regressive tax exert an equalizing force?

For the reduction in inequality to be higher with the tax than without it, the following condition must hold:

$$\boldsymbol{RE_N = \frac{(1-g)RE_t + (1+b)RE_B}{1-g+b} > RE_B}$$

Condition 2

$$\boldsymbol{\rightarrow RE_t > -\frac{(g)}{(1-g)} RE_B}$$

Is a tax equalizing?
Answer for a system with a tax and a transfer

		System with a Transfer that is		
		Regressive $\rho_B^K < 0$	**Neutral** $\rho_B^K = 0$	**Progressive** $\rho_B^K > 0$
Adding a Tax that is	**Regressive** $\Pi_T^K < 0$	Always More Unequalizing	Always Unequalizing	More Equalizing only if Condition 2 holds
	Neutral $\Pi_T^K = 0$	Always More Unequalizing	No Change in Inequality	Always More Equalizing
	Progressive $\Pi_T^K > 0$	More Equalizing only if Condition 2 holds	Always Equalizing	Always More Equalizing

Condition 2

$$\rightarrow RE_t > -\frac{(g)}{(1-g)} RE_B$$

Equalizing Regressive Taxes Exist in Real Life

- The US and the UK had regressive equalizing taxes in the past (O'Higgins & Ruggles, 1981 and Ruggles & O'Higgins, 1981)
- Chile's 1996 fiscal system had equalizing regressive taxes (Engel et al., 1999)
    - Redistributive Effect of Net Fiscal System (taxes and transfers together = 0.0583 (decline in Gini points)
    - Redistributive Effect of System with Taxes only = - 0.0076
    - Redistributive Effect of System with Transfers but without Taxes = 0.0574
- ➢ Note that 0.0583 > 0.0574
- CEQs for Chile 2009 and South Africa 2010 also show that regressive consumption taxes are equalizing

Some Results…

	Brazil	Chile[a]	Colombia	Indonesia[b]	Mexico	Peru	South Africa[c]	Average
Marginal Contributions								
From Market to Post-fiscal Income								
Redistributive Effect	0.0446	0.0370	0.0073	0.0061	0.0308	0.0151	0.0789	0.0306
Direct taxes	0.0171	0.0179	0.0019		0.0140	0.0060	0.0311	0.0125
Direct transfers	0.0382	0.0220	0.0057	0.0043	0.0113	0.0048	0.0711	0.0207
Indirect taxes	-0.0014	0.0027	-0.0017	-0.0028	0.0027	0.0052	0.00001	0.0007
Indirect subsidies	0.0008	0.0004	0.0015	0.0052	0.0047			0.0025
Kakwani[d]								
Direct taxes	0.1738	0.3481	0.1373		0.2411	0.3853	0.1109	0.2328
Direct transfers	0.5310	0.9064	0.9233	0.6248	0.7931	0.9612	0.9955	0.8193
Indirect taxes	-0.0536	-0.0172	-0.1986	-0.0513	0.0129	0.0527	-0.0712	-0.0466
Indirect subsidies	0.8295	0.7978	0.5034	0.0645	0.2457			0.4882

Source: author's calculations based on Brazil: Higgins and Pereira, 2014; Chile: Jaime Ruiz Tagle and Dante Contreras, 2014; Colombia: Melendez, 2014; Indonesia: Jellema et al., 2014; Mexico: Scott, 2014; Peru: Jaramillo, 2013; South Africa: Inchauste et al., 2014.

Generalizing the result to n taxes and m transfers

Is a particular tax or transfer equalizing?

- The results shown above can be generalized to *n* taxes and *m* transfers (in chapter but not presented here)

- Note that the results do not require for the size of total taxes and total transfers to be the same (see conditions 1 and 2 above)

Path Dependency Underscores the Importance of the Analysis Being Comprehensive

- Obvious reason
  - To capture the full effect of the net fiscal system

- More subtle but fundamental reason
  - ➢ Assessing the progressivity of a tax or a transfer in isolation can give the wrong answer to the question: Is the tax or the transfer equalizing?
  - ➢ Think of the example of Chile and South Africa just shown above

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What is the contribution of a particular tax or transfer to the change in inequality?

- Sequential method
  - May give the wrong answer to the “without vs. with comparison” because it ignores path dependency
- **Marginal contribution method (same for poverty)**
  - Gives correct answer to the “without vs. with comparison” but does not fulfill the principle of aggregation: i.e., the sum of the marginal contributions will not equal the total change in inequality (except by coincidence)
- Average Contribution with all possible paths considered (Shapley value)
  - Fulfills the principle of aggregation, takes care of path dependency but the sign may be different from the marginal contribution => problematic?

Calculating the Marginal Contribution of a Tax

The marginal contribution of a tax is defined as

$$MC_t = G_{x+B} - G_{x+B-t}$$

Where G_{x+B}, G_{x+B-t} and are the Gini coefficient of income with the transfer but **without** the tax and the Gini coefficient with the transfer and **with** the tax, respectively

If MC_t **> 0,** remember, the tax is equalizing

Sequential vs. Marginal Contribution
Why the sequential method can be misleading

Chile's 1996 fiscal system (Engel et al., 1999)

- Sequential contribution method: -0.0076
- Marginal contribution method: 0.009

Fiscal Policy and Inequality
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Progressivity vs. Size of Intervention: A System with One Tax and One Transfer

- In a system with one tax and one transfer:

$$MC_T = G_{X+B} - G_{X-T+B} = \dots = \frac{g\prod_T^K + b\rho_B^K}{1-g+b} - \frac{b}{1+b}\rho_B^K$$

- Getting the partial derivatives:

$$\frac{\partial MC_T}{\partial g} = \frac{(1+b)\prod_T^K + b\rho_B^K}{(1-g+b)^2}$$

$$\frac{\partial MC_T}{\partial \prod_T^K} = \frac{g}{1-g+b}$$

Effectiveness: previous CEQ index

- In Lustig and Higgins (2013) effectiveness is defined as:

$$\frac{\Delta Gini}{Spending / GDP}$$

  - While this indicator would correctly rank fiscal incidences with positive contribution to reducing inequality, it has an awkward interpretation.
  - It can be interpreted as how much Gini index would change if the tax or transfer of interest is scaled up to the size of GDP using a linear extrapolation. As a result, the change in Gini could exceed unity (maximum possible value)

Effectiveness: new CEQ indices

- Moreover, the effectiveness indicators usually rely on an "ideal" value as the reference point which the previous index lacked such reference point.

- Therefore, in the new handbook we define three new indicators to account for these shortcomings:
  1. **Impact Effectiveness**
  2. **Spending Effectiveness**
  3. **Impact-Ranked Effectiveness**

Effectiveness: Impact Effectiveness

$$\boldsymbol{Impact\ Effectiveness_{T\ (or\ B)}^{End\ income} = \frac{MC_{T\ (or\ B)}^{End\ income}}{MC_{T\ (or\ B)}^{End\ income^{*}}} * 100\%}$$

where $\mathrm{MC_{T\ (or\ B)}^{End\ income}}$ is the marginal contribution of a Tax (or a Benefit) to reducing inequality or poverty and $\mathrm{MC_{T\ (or\ B)}^{End\ income^{*}}}$ is the maximum possible $\mathrm{MC_{T\ (or\ B)}^{End\ income}}$ if the same amount of Tax (or Benefit) is distributed differently among individuals

Effectiveness: Spending Effectiveness

$$\boldsymbol{Spending\ Effectiveness}_{\boldsymbol{T\ (or\ B)}}^{\boldsymbol{End\ income}} = \frac{\boldsymbol{T^{*}\ (or\ B^{*})}}{\boldsymbol{T\ (or\ B)}} * \mathbf{100\%}$$

where $\mathrm{T}^{*}\ (\mathrm{or\ B}^{*})$ is the minimum amount of T (or B) that is needed to create the same $\mathrm{MC}_{\mathrm{T\ (or\ B)}}^{\mathrm{End\ income}}$.

Effectiveness: Impact-Ranked Effectiveness

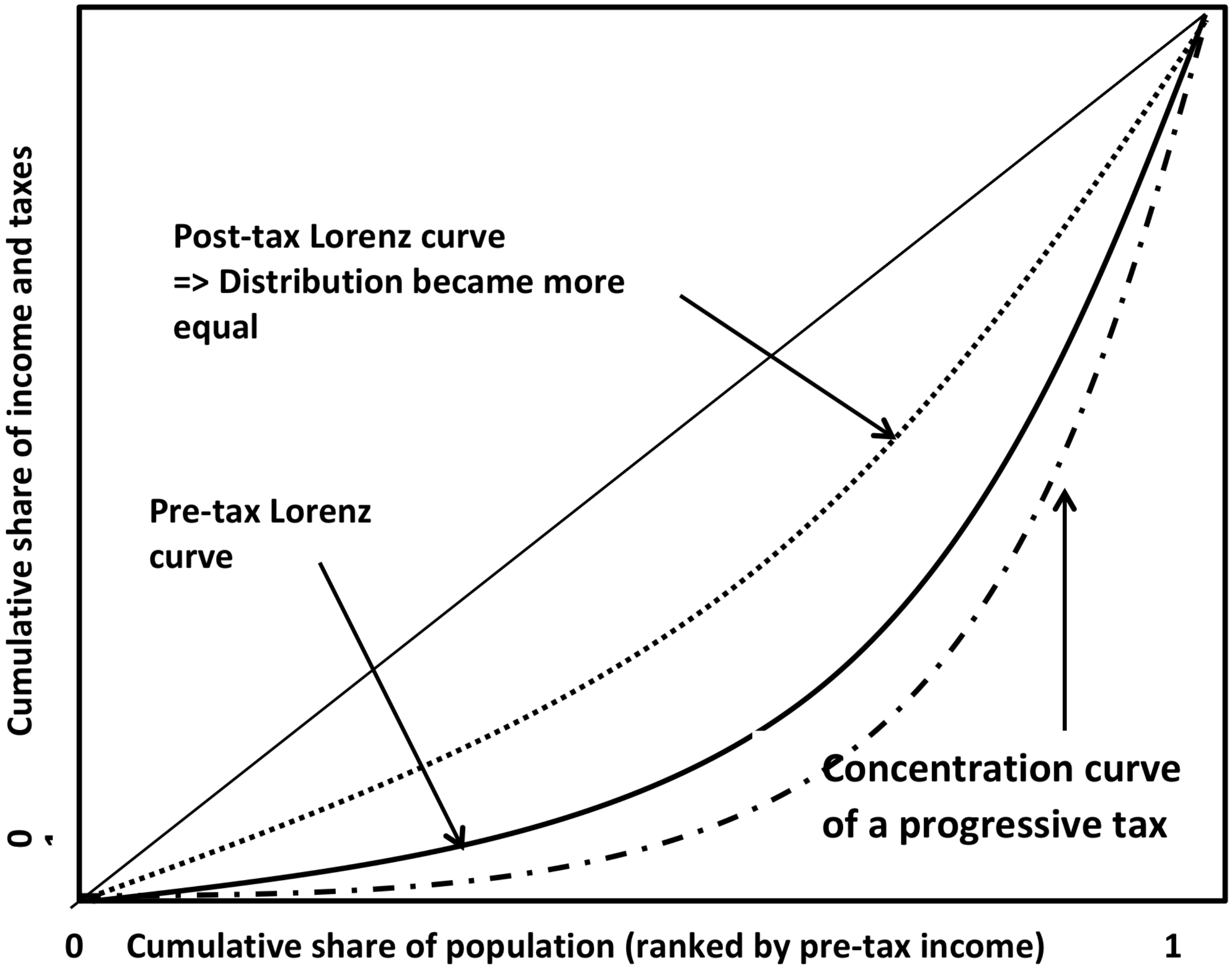
$$\boldsymbol{IRE_{T\ (or\ B)}^{End\ income} = Rank\left\{\left(\frac{1 - MC_{T\ (or\ B)}^{End\ income^{*}}}{1 - MC_{T\ (or\ B)}^{End\ income}} * Sign\left(MC_{T\ (or\ B)}^{End\ income}\right)\right)^{Sign\left(MC_{T\ (or\ B)}^{End\ income}\right)}\right\}}$$

References

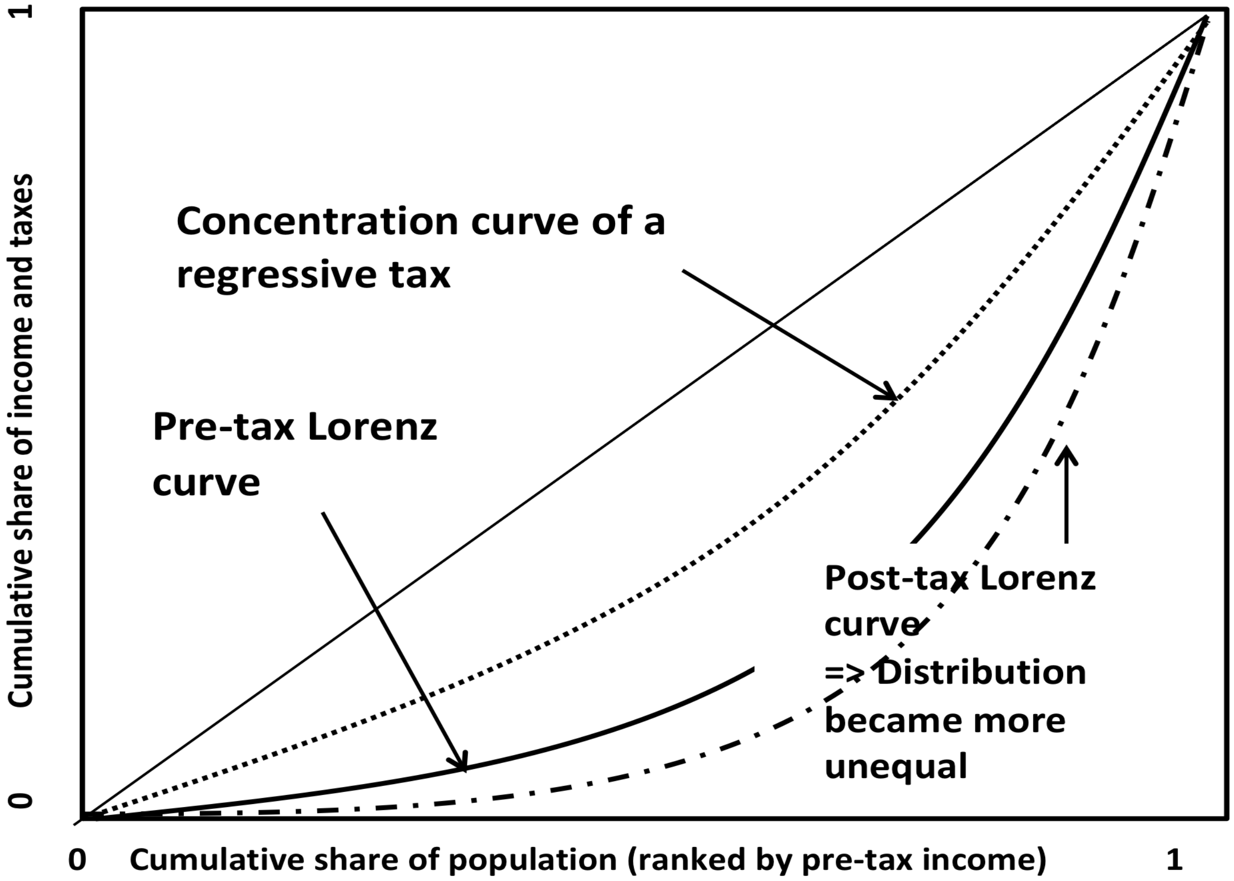
- Duclos, Jean-Yves and Abdelkrim Araar. 2007. *Poverty and Equity: Measurement, Policy and Estimation with DAD* (Vol. 2). Springer. Chapters 7 and 8. (available online)
- Lambert, Peter J. (2001). *The Distribution and Redistribution of Income: A Mathematical Analysis*. Manchester University Press. Third Edition. Chapter 11. (not available online)

APPENDIX

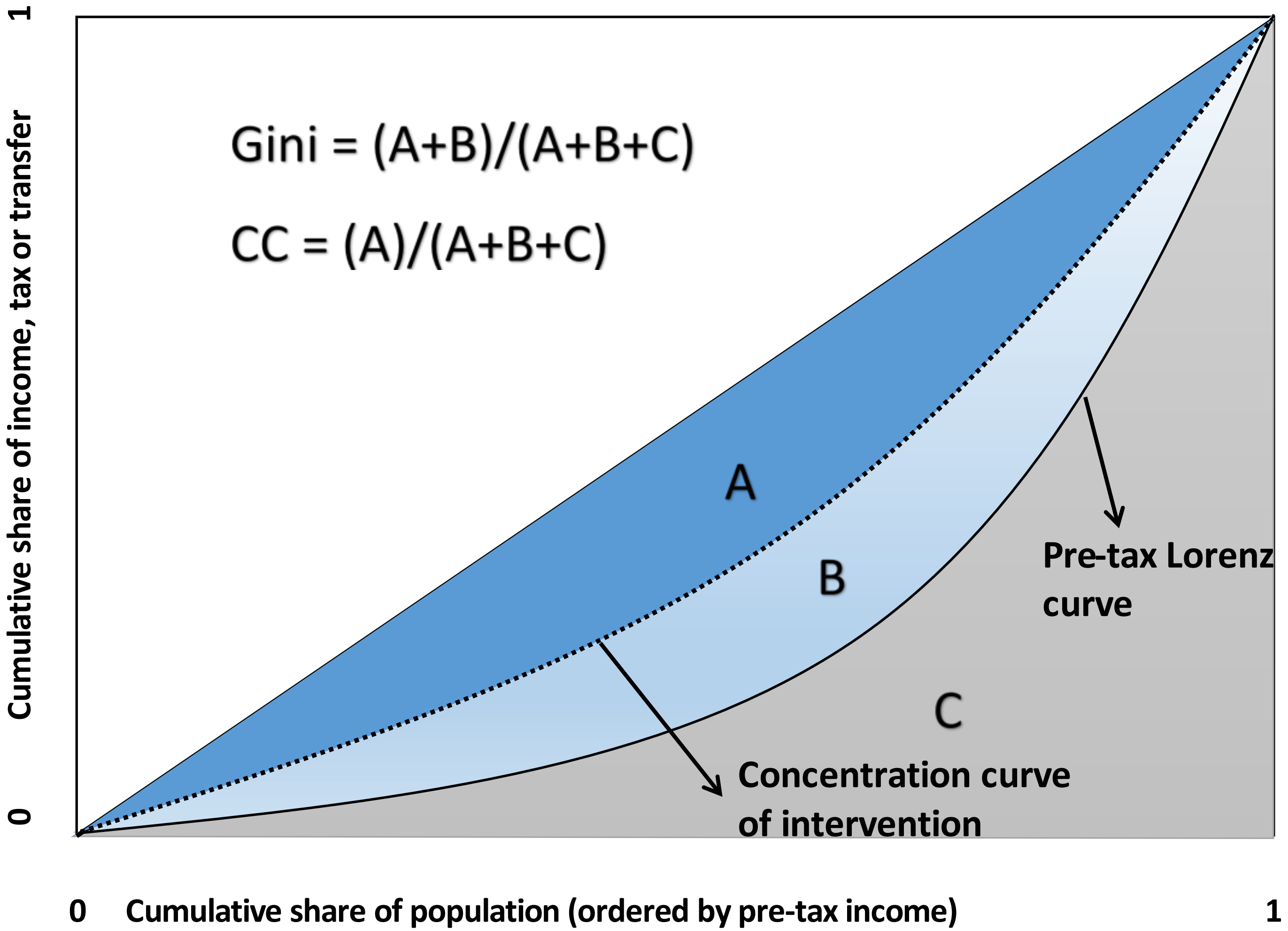
Concentration Curve Progressive Tax



Concentration Curve Regressive Tax



Concentration Coefficient: CC



Gini = (A+B)/(A+B+C)

CC = (A)/(A+B+C)

Pre-tax Lorenz curve

Concentration curve of intervention

Kakwani Index: Tax

The Kakwani index of progressivity of a tax ***t*** is defined as:

$$\prod_{T}^{K} = \mathbf{C_t - G_x}$$

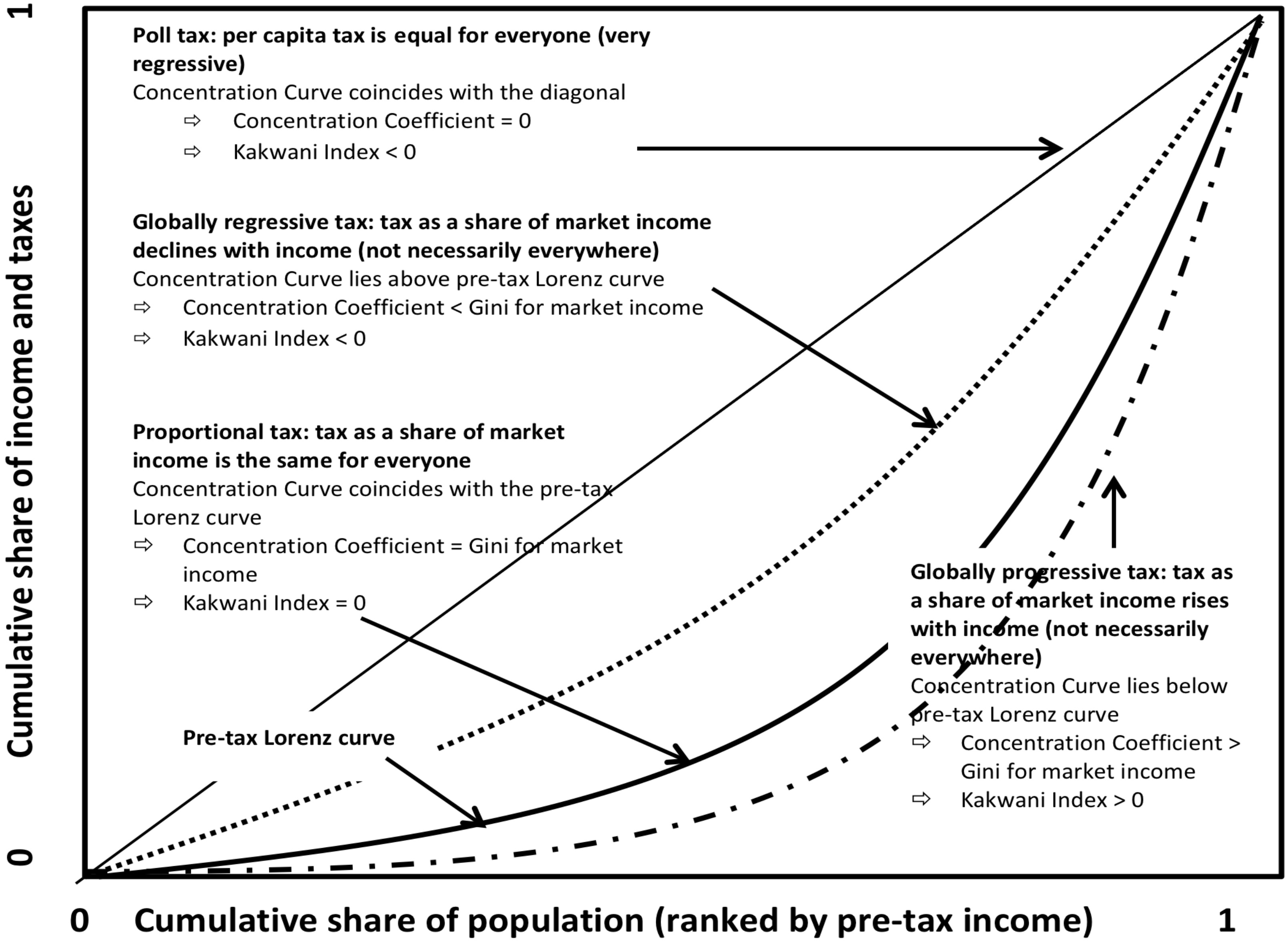
Where:

- $\mathbf{G_x}$ is the Gini coefficient of pre-tax income
- $\mathbf{C_t}$ is the concentration coefficient of the tax ***t***

Kakwani Index

- Progressive Tax: $\prod_{T}^{K} = C_t - G_x > 0$
- Proportional Tax: $\prod_{T}^{K} = C_t - G_x = 0$
- Regressive Tax: $\prod_{T}^{K} = C_t - G_x < 0$

Progressivity of Taxes: A Diagrammatic Representation



Poll tax: per capita tax is equal for everyone (very regressive)
Concentration Curve coincides with the diagonal
- ⇨ Concentration Coefficient = 0
- ⇨ Kakwani Index < 0

Globally regressive tax: tax as a share of market income declines with income (not necessarily everywhere)
Concentration Curve lies above pre-tax Lorenz curve
- ⇨ Concentration Coefficient < Gini for market income
- ⇨ Kakwani Index < 0

Proportional tax: tax as a share of market income is the same for everyone
Concentration Curve coincides with the pre-tax Lorenz curve
- ⇨ Concentration Coefficient = Gini for market income
- ⇨ Kakwani Index = 0

Pre-tax Lorenz curve

Globally progressive tax: tax as a share of market income rises with income (not necessarily everywhere)
Concentration Curve lies below pre-tax Lorenz curve
- ⇨ Concentration Coefficient > Gini for market income
- ⇨ Kakwani Index > 0

In a world with just a *single* tax

- A necessary and sufficient condition for a tax to be equalizing is to have a positive Kakwani index
- A necessary and sufficient condition for a tax to be unequalizing is to have a negative Kakwani index
- Analogous conditions apply to transfers

Kakwani Index: Transfer

The Kakwani index of progressivity of a transfer **B** is defined as:

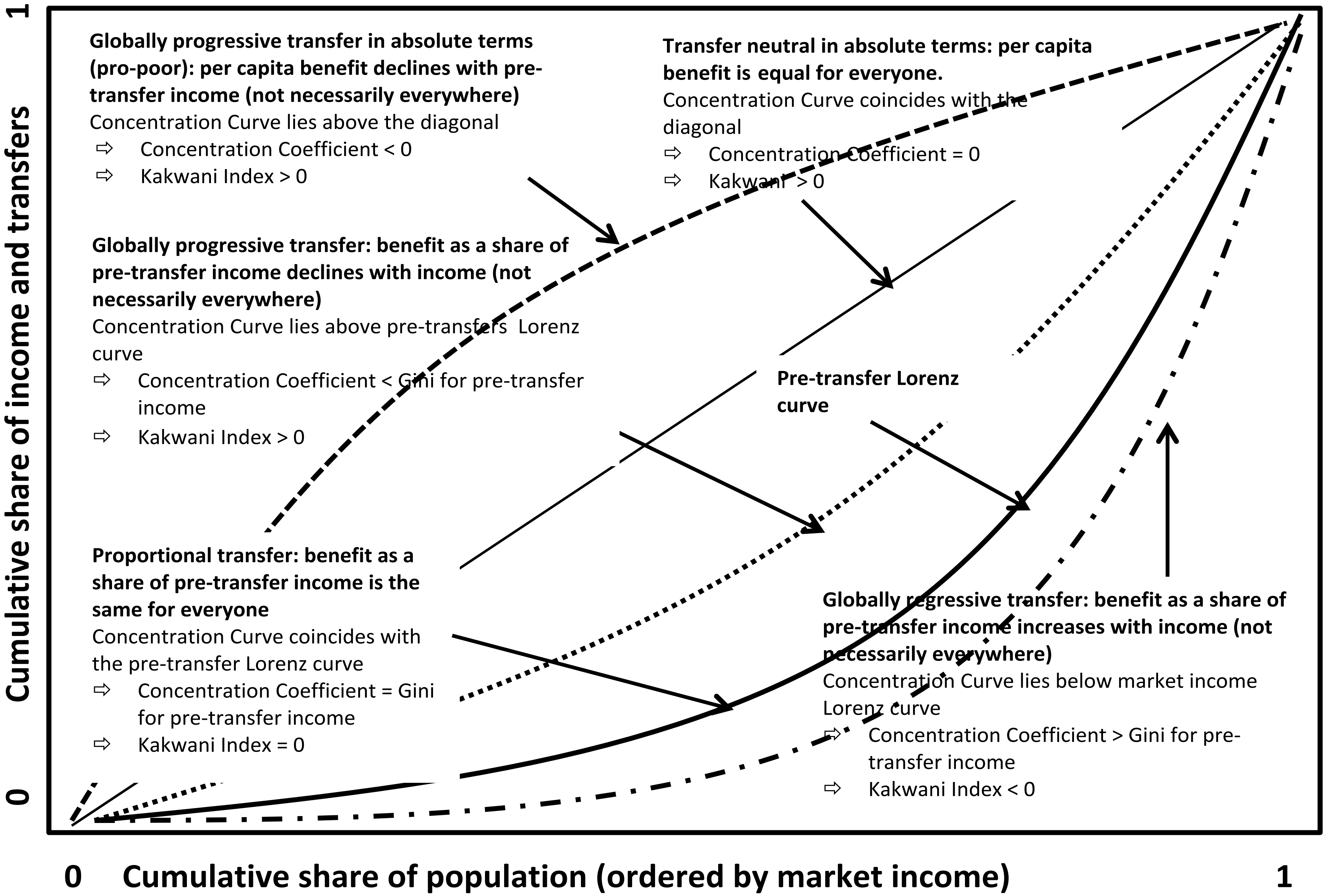
$$\rho_B^K = \mathbf{G_X - C_B}$$

Where:

- **G_X** is the Gini coefficient of pre-transfer income
- **C_B** is the concentration coefficient of the transfer **B**

➢ Note that the Gini coefficient and the concentration coefficient are in reversed order from the Kakwani index for a tax

Progressivity of Transfers: A Diagrammatic Representation



Globally progressive transfer in absolute terms (pro-poor): per capita benefit declines with pre-transfer income (not necessarily everywhere)

Concentration Curve lies above the diagonal

- ⇨ Concentration Coefficient < 0
- ⇨ Kakwani Index > 0

Transfer neutral in absolute terms: per capita benefit is equal for everyone.

Concentration Curve coincides with the diagonal

- ⇨ Concentration Coefficient = 0
- ⇨ Kakwani > 0

Globally progressive transfer: benefit as a share of pre-transfer income declines with income (not necessarily everywhere)

Concentration Curve lies above pre-transfers Lorenz curve

- ⇨ Concentration Coefficient < Gini for pre-transfer income
- ⇨ Kakwani Index > 0

Pre-transfer Lorenz curve

Proportional transfer: benefit as a share of pre-transfer income is the same for everyone

Concentration Curve coincides with the pre-transfer Lorenz curve

- ⇨ Concentration Coefficient = Gini for pre-transfer income
- ⇨ Kakwani Index = 0

Globally regressive transfer: benefit as a share of pre-transfer income increases with income (not necessarily everywhere)

Concentration Curve lies below market income Lorenz curve

- ⇨ Concentration Coefficient > Gini for pre-transfer income
- ⇨ Kakwani Index < 0

Impact on Inequality Depends On…

- Progressivity of a tax (transfer)
- Size of the tax (transfer), where size equals the total tax (transfer) divided by total pre-tax (pre-transfer) income

➢ A large regressive tax can be more equalizing than a small progressive one as shown in next slide

Redistributive Effect and the Progressivity and Level of Taxes

	Gross Income		Tax A=50.5%		Net Income under A		Tax B=1%		Net Income under B	
	Income	Distribution	Tax	Distribution	Income	Distribution	Tax	Distribution	Income	Distribution
1	21	21%	1	2%	20	40%	0	0%	21	21%
2	80	79%	50	98%	30	60%	1	100%	79	79%
Total	101	100%	51	100%	50	100%	1	100%	100	100%

Source: Duclos and Tabi, 1996, Table 1.

Progressivity vs. Size of Intervention: A System with Only One Transfer

- In a system with only one tax:

$$MC_B = RE_B = \frac{b}{1+b}\rho_B^K$$

- Getting the partial derivatives:

$$\frac{\partial MC_B}{\partial b} = \frac{1}{(1+b)^2}\rho_B^K$$

$$\frac{\partial MC_B}{\partial \rho_B^K} = \frac{b}{1+b}$$

- For a change in progressivity to be more equalizing than a change in size :

$$\frac{\partial MC_B}{\partial \rho_B^K} > \frac{\partial MC_B}{\partial b} \Rightarrow b(1+b) > \rho_B^K$$

Progressivity vs. Size of Intervention: A System with Only One Tax

- In a system with only one tax:

$$RE_T = \frac{g}{1-g}\prod_T^K$$

- Getting the partial derivatives:

$$\frac{\partial RE_T}{\partial g} = \frac{1}{(1-g)^2}\prod_T^K$$

$$\frac{\partial RE_T}{\partial \prod_T^K} = \frac{g}{1-g}$$

- For a change in progressivity to be more equalizing than a change in size:

$$\frac{\partial RE_T}{\partial \prod_T^K} > \frac{\partial RE_T}{\partial g} \Rightarrow g(1-g) > \prod_T^K$$

Is a transfer equalizing?
Answer for a system with a tax and a transfer

		Adding a Transfer that is		
		Regressive $\rho_B^K < 0$	**Neutral** $\rho_B^K = 0$	**Progressive** $\rho_B^K > 0$
System with a Tax that is	**Regressive** $\Pi_T^K < 0$	Less Unequalizing if and only if Condition 3 holds	Always Less Unequalizing	Always Less Unequalizing
	Neutral $\Pi_T^K = 0$	Always Unequalizing	No Change in Equality	Always Equalizing
	Progressive $\Pi_T^K > 0$	Always Less Equalizing	Always Less Equalizing	More Equalizing if and only if Condition 3 holds

Condition 3 $$RE_B > \frac{b}{1+b} RE_T$$

Progressivity vs. Size of Intervention: A System with One Tax and One Transfer

- For a change in progressivity to be more equalizing than a change in size:

$$\frac{\partial MC_T}{\partial \prod_T^K} > \frac{\partial MC_T}{\partial g} \Rightarrow g > \prod_T^K + RE_N$$

- Similarly:

$$\frac{\partial MC_B}{\partial \rho_B^K} > \frac{\partial MC_B}{\partial b} \Rightarrow b > \rho_B^K - RE_N$$

Progressivity vs. Size of Intervention: A System with Multiple Taxes and Transfers

- The formulas are the same:

$$\frac{\partial MC_{T_i}}{\partial \Pi^K_{T_i}} > \frac{\partial MC_{T_i}}{\partial g_i} \Rightarrow g_i > \Pi^K_{T_i} + RE_N$$

$$\frac{\partial MC_{B_j}}{\partial \rho^K_{B_j}} > \frac{\partial MC_{B_j}}{\partial b_j} \Rightarrow b_j > \rho^K_{B_j} - RE_N$$

Next Steps: Relaxing Assumptions

- **Reranking**: individuals can swap positions in the post-fiscal income ordering; true of all systems in the real world
- **No dominance:** post-fiscal Lorenz curve crosses the pre-fiscal Lorenz curve; normative parameter must be explicitly introduced (will not be covered today)
- **Different pre-fiscal (original) distributions:** comparing the inequality- and poverty-reducing capacity of fiscal systems across countries and over time (will not be covered today)

Reranking: Introduction

- **In the presence of reranking, the usual rule of thumbs do not work properly.** For example, a progressive tax can be uneqaulizing

	Individual	Original Income	Tax (% Income)	End Income
	1	**10**	**0** (0%)	**10**
	2	**11**	**2** (18.18%)	**9**
	3	**12**	**4** (33.33%)	**8**
	4	**13**	**6** (46.15%)	**7**
Total	**-**	**46**	**12**	**34**
Average	**-**	**11.5**	**3**	**8.5**
Gini	**-**	**0.054**	**-**	**0.074**

Reranking: Defining a new progressivity index (1)

- **Calculating the progressivity with respect to any pre-tax (or pre-transfer) income concept suffers from the same shortcoming.** So it doesn't matter whether we use the original income (i.e., pre-all taxes and transfers) or the "Final income without a specific tax (or transfer)", the progressivity index does not give us a clear answer about the equalizing effect of a tax (or transfer).
- **Calculating the progressivity with respect to the Final income (i.e., post-all taxes and transfers) creates <u>complete dependence</u> between the indices of taxes and transfers.** That means, for example, if you change a tax, the progressivity of a transfer will change too!

Reranking: Defining a new progressivity index (2)

- **The middle ground is to define a semi-independent index of progressivity.** We suggest to calculate the progressivity index using the monetary values of the Original Income and a specific tax (or transfer) and the ranking of individuals with respect to the End Income. In this way, unless a change in a tax or transfer changes the End Income ranking, the progressivity indices of taxes and transfer will be independent.

Reranking: Defining a new progressivity index (3)

- Formally, we define this **modified Kakwani index of a tax** as follows:

Modified Kakwani index of a tax w.r.t. the End Income ranking

$$\Pi_T^{K^{EI}} = C_T^{EI} - C_{OI}^{EI}$$

Concentration coefficient of a tax w.r.t. the End Income ranking

Concentration coefficient of the Original Income w.r.t. the End Income ranking

- and for a **transfer**:

$$\rho_B^{K^{EI}} = C_{OI}^{EI} - C_B^{EI}$$

- where:

$$C_Y^Q = \frac{2Cov(Y, F_Q)}{Avg(Y)}$$

Normalized rank with respect to the income Q

Reranking: Does adding a tax to a system with a transfer in place increase the equality?

- The new index can produce general rule of thumbs. An example is follows

		To a system with a Transfer that with respect to the end income ranking is		
		Regressive $\rho_B^{K^{X-T+B}} < 0$	Neutral $\rho_B^{K^{X-T+B}} = 0$	Progressive $\rho_B^{K^{X-T+B}} > 0$
Adding a Tax that with respect to the end income ranking is	Regressive $\Pi_T^{K^{X-T+B}} < 0$	More Equalizing if and only if condition A holds	More Equalizing if and only if condition A holds	More Equalizing if and only if condition A holds
	Neutral $\Pi_T^{K^{X-T+B}} = 0$	More Equalizing if and only if condition A holds	More Equalizing if and only if condition A holds	Always Equalizing
	Progressive $\Pi_T^{K^{X-T+B}} > 0$	More Equalizing if and only if condition A holds	Always Equalizing	Always Equalizing

$$\left(\frac{g\Pi_T^{K^{X-T+B}} + \frac{gb}{1+b}\rho_B^{K^{X-T+B}}}{1-g+b}\right) + \left(G_{X+B} - C_{X+B}^{X-T+B}\right) > 0 \quad \textbf{(A)}$$

Thank you!